

# Satisfiability Checking of Temporal Logics for Verification of Pointer Systems

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# Background

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Background:

- State Explosion Problem, especially in software model checking.
- Abstraction is a key issue.
- Various existing tools.
  - Bandera (Java), SLAM (C), Blast (C), ....
- Few handle properties on structures built up by pointers.
- Hagiya-Takahashi proposed a method of abstraction using temporal formula.

Goal: A tool that supports abstraction to make model checking on pointer systems feasible.

Predicate Abstraction

Pointer System

Satisfiability Checking

Hybrid Temporal Logic

# Predicate Abstraction

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## Summary:

- A method to build an abstract Kripke structure from a given (possibly infinite) concrete Kripke structure.
- It requires:
  - a set of **predicates**,
  - a procedure to calculate a **precondition** of a predicate, and
  - a **decision procedure** for satisfiability of predicates.
- The resulting abstraction is sound for ACTL\* properties.

# Predicate Abstraction (cont)

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AP: the set of atomic propositions

Kripke structure:  $\mathcal{C} = (S, R, L)$

- $S$ : the set of states
- $R \subseteq S \times S$ : the transition relation
- $L : AP \rightarrow \mathcal{P}(S)$ : the labelling function

$Q := \{Q_i \mid i = 1, \dots, n\}$ : a set of **predicates** on  $S$ . i.e. for each  $s \in S$ , either  $Q_i$  is true or false at  $s$ .

Predicate  $Q$  is a **precondition** of predicate  $Q'$  if

$$(s, s') \in R, Q' \text{ is true at } s' \implies Q \text{ is true at } s.$$

A **decision procedure** DP for satisfiability checking of predicates should behave as follows:

- If  $Q$  is satisfiable, i.e.  $\exists s \in S. Q$  is true at  $s$ , then DP should reply “yes” ,
- otherwise, DP should reply “yes” or “no” .

Suppose that from predicate  $Q$  we can compute one of its precondition. Let  $\text{pre}(Q)$  be the result.

# Abstract Transition System

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The abstract transition system  $(A, \bar{R})$  is defined as follows:

- $A := \mathcal{P}(\mathcal{Q})$
- For  $a, a' \in A$ ,  $(a, a') \in \bar{R} \iff$  DP replies “yes” for  $\bigwedge\{Q \mid Q \in a\} \wedge \bigwedge\{\neg Q \mid Q \in \mathcal{Q} \setminus a\} \wedge \bigwedge\{\text{pre}(Q') \mid Q' \in a'\} \wedge \bigwedge\{\text{pre}(\neg Q') \mid Q' \in \mathcal{Q} \setminus a'\}$

Mapping  $\beta : S \rightarrow A$  can be defined naturally:

$$\beta(s) = \{Q \in \mathcal{Q} \mid Q \text{ is true at } s\}$$

$\varphi_0$ : an ACTL\* formula

$AP'$ : the set of atomic propositions appeared in  $\varphi_0$ .

$Q_p$ : predicate “ $s \in L(p)$ ”, for  $p \in AP'$ .

Suppose for each  $p \in AP'$ ,  $Q_p \in \mathcal{Q}$ . Then  $(A, \bar{R})$  is naturally extended to a Kripke structure  $\mathcal{A} = (A, \bar{R}, \bar{L})$ :

$$a \in \bar{L}(p) \iff p \in AP' \text{ and } Q_p \in a.$$

## Theorem

$\mathcal{A}$  is a sound abstraction: for  $s \in S$ ,

$$\mathcal{A}, \beta(s) \models \varphi_0 \implies \mathcal{C}, s \models \varphi_0,$$

hence

$$\mathcal{A} \models \varphi_0 \implies \mathcal{C} \models \varphi_0.$$



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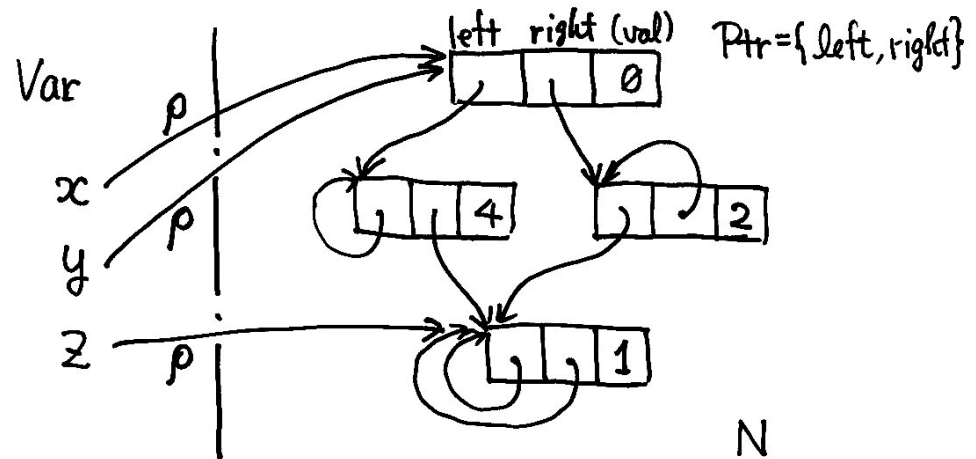
# Pointer Structure

**Var**: the set of variables

**Ptr**: the set of (names of) pointers

**pointer structure**: tuple  $(N, p, v, \rho)$  s.t.

- $N$  is the set of nodes.
- $p : \text{Ptr} \rightarrow N \rightarrow N$
- $v : N \rightarrow \omega$
- $\rho : \text{Var} \rightarrow N$



# Program

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**statement:** one of the following, possibly with a **label**

- $\text{if } (cond) \text{ goto } label$
- $x := y$
- $x := y.p$
- $x := \text{new}()$
- $x.val := n$
- $x.p := y$

where  $x, y \in \text{Var}$ ;  $n \in \omega$ ;  $p \in \text{Ptr}$ .

$cond$  is  $x == y$ ,  $x.val == n$ , or their boolean combination.

**program:** finite sequence of statements

# Transition system

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**PtrStr** := the class of all pointer structures

$\text{PtrStr} \times \omega$  can be regarded as a transition system by a program  $P$ :

$(S, i) \rightarrow (S', i') \stackrel{\text{def}}{\iff}$  “Executing the  $i$ -th statement of  $P$  on  $S$ , the resulting structure is  $S'$  and the next to be executed is the  $i'$ -th statement.”

This is the target system on which we will define predicates, calculate preconditions, and find a decision procedure for satisfiability.

# Two-way CTL

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Mod: the set of modalities

For  $a \in \text{Mod}$ ,  $\bar{a} \in \text{Mod}$ ,  $\bar{\bar{a}} = a$

two-way CTL formula  $\varphi$ :

$\varphi = p \mid \neg p \mid \varphi \vee \varphi \mid \varphi \wedge \varphi \mid \mathbf{E}_A \mathbf{X} \varphi \mid \mathbf{A}_A \mathbf{X} \varphi$   
 $\mid \mathbf{E}_A \mathbf{F} \varphi \mid \mathbf{A}_A \mathbf{F} \varphi \mid \mathbf{E}_A \mathbf{G} \varphi \mid \mathbf{A}_A \mathbf{G} \varphi$

( $p \in \text{AP}$ ,  $A \subseteq \text{Mod}$ ,  $0 < |A| < \omega$ )

Form := the set of two-way CTL formulas.

Kripke structure  $K = (S, R, L)$ .

- $R : \text{Mod} \rightarrow \mathcal{P}(S \times S)$
- $(s, s') \in R(a) \iff (s', s) \in R(\bar{a})$
- $L : \text{AP} \rightarrow \mathcal{P}(S)$

# Satisfaction Relation

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- For formula  $\varphi$  and  $s \in S$ ,
  - $s \models p \iff s \in L(p)$
  - $s \models \neg p \iff s \notin L(p)$
  - $s \models \varphi_1 \vee \varphi_2 \iff s \models \varphi_1 \text{ or } s \models \varphi_2$
  - $s \models \varphi_1 \wedge \varphi_2 \iff s \models \varphi_1 \text{ and } s \models \varphi_2$
  - $s \models \mathbf{E}_A \mathbf{X} \varphi \iff \exists s' \in S, \exists a \in A. (s, s') \in R_a, s' \models \varphi.$
  - $s \models \mathbf{A}_A \mathbf{X} \varphi \iff$   
 $\quad \forall s' \in S, \forall a \in A. (s, s') \in R_a \implies s' \models \varphi.$
  - $s \models \mathbf{E}_A \mathbf{F} \varphi \iff \exists \sigma: A\text{-path}. \sigma_0 = s, \exists i. \sigma_i \models \varphi$
  - $s \models \mathbf{A}_A \mathbf{F} \varphi \iff \forall \sigma: A\text{-path}. \sigma_0 = s \implies \exists i. \sigma_i \models \varphi$
  - $s \models \mathbf{E}_A \mathbf{G} \varphi \iff \exists \sigma: A\text{-path}. \sigma_0 = s, \forall i. \sigma_i \models \varphi$
  - $s \models \mathbf{A}_A \mathbf{G} \varphi \iff \forall \sigma: A\text{-path}. \sigma_0 = s \implies \forall i. \sigma_i \models \varphi$

# Formula and Kripke Structure for Pointer Structure

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$AP := \text{Var} \cup \omega$ ,  $\text{Mod} := \text{Ptr} \cup \{\bar{a} \mid a \in \text{Ptr}\}$

Formula Examples:  $(x, y, z, \text{nil} \in \text{Var}; a \in \text{Ptr})$

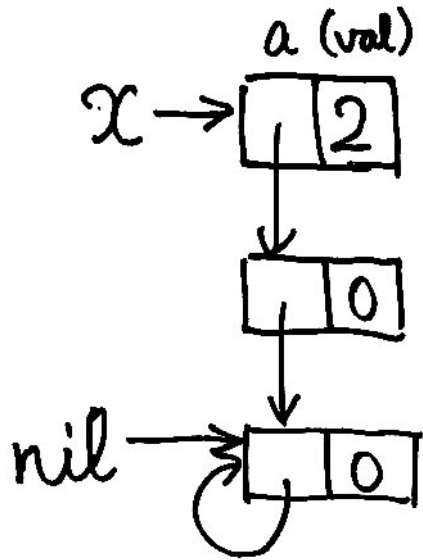
- $x \wedge 2 \wedge \mathbf{E}_a \mathbf{F} \text{nil}$
- $x \wedge \mathbf{E}_a \mathbf{X} \mathbf{E}_a \mathbf{F} x$
- $z \wedge \mathbf{E}_{\bar{a}} \mathbf{X} x \wedge \mathbf{E}_{\bar{a}} \mathbf{X} y$

We can naturally define a Kripke structure  $K(S) = (N, R, L)$  from a pointer structure  $S = (N, p, v, \rho)$ :

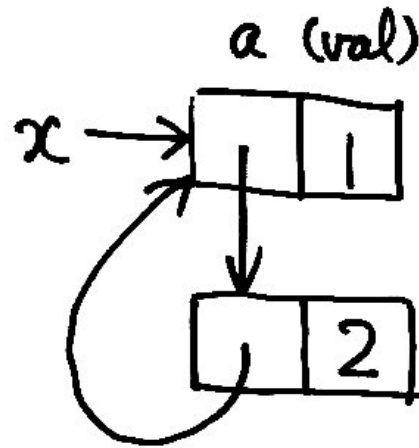
- $R(a) = \{(n, n') \mid p(a)(n) = n'\}$
- $R(\bar{a}) = \{(n', n) \mid p(a)(n) = n'\}$
- $L(x) = \{\rho(x)\} \quad (x \in \text{Var})$
- $L(m) = \{n \in N \mid v(n) = m\} \quad (m \in \omega)$

# Examples:

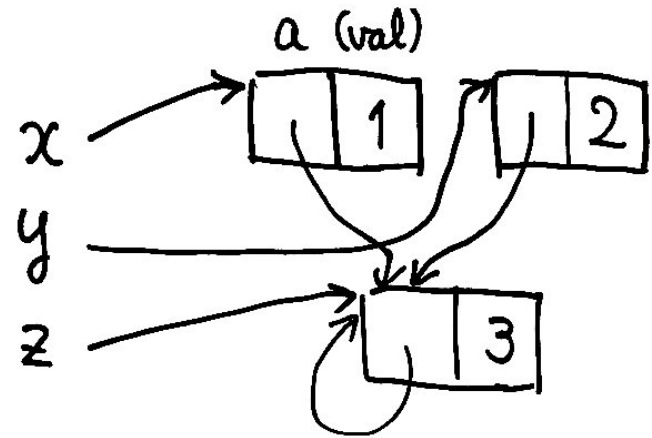
$$x \wedge 2 \wedge \mathbf{E}_a \mathbf{F} \text{ nil}$$



$$x \wedge \mathbf{E}_a \mathbf{X} \mathbf{E}_a \mathbf{F} x$$



$$z \wedge \mathbf{E}_{\bar{a}} \mathbf{X} x \wedge \mathbf{E}_{\bar{a}} \mathbf{X} y$$





# Predicate

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We define **predicates** for the abstraction:

1.  $\exists\varphi$  is a predicate for a formula  $\varphi$ .
2.  $\neg Q_1$  and  $Q_1 \vee Q_2$  are predicates for predicates  $Q_1$  and  $Q_2$ .

For a Kripke structure  $K$  and a predicate  $Q$ , we define

$K \models^* Q$ :

- $K \models^* \exists\varphi \iff$  There exists  $n \in N$  s.t.  $K, n \models \varphi$ .
- $K \models^* \neg Q \iff K \not\models^* Q$ .
- $K \models^* Q_1 \vee Q_2 \iff K \models^* Q_1$  or  $K \models^* Q_2$ .

For a pointer structure  $S$  and a predicate  $Q$ ,

$$S \models^* Q \stackrel{\text{def}}{\iff} K(S) \models^* Q$$

Abbrev:  $Q_1 \wedge Q_2$ ,  $Q_1 \rightarrow Q_2$ ,  $Q_1 \leftrightarrow Q_2$ ,  $\forall\varphi (= \neg\exists\neg\varphi)$

# Precondition

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## Theorem

(1) When  $P_i$  is “if  $c$  goto  $l$ ,” there is a procedure to construct a predicate  $Q$  from  $P_i$  such that

$$c \text{ holds on } S \iff S \models^* Q.$$

(2) When  $P_i$  is not an if-statement, there is a procedure to construct a predicate  $Q$  from a given predicate  $Q'$  such that for any  $S, S' \in \text{PtrStr}$  with  $(S, i) \rightarrow_P (S', i + 1)$ :

$$S \models^* Q \iff S' \models^* Q'.$$

So far we have proved it only for (one-way) CTL.

# Decision Procedure

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We need a decision procedure DP such that for a given predicate  $Q$ ,

- When  $Q$  is satisfied by a pointer structure, it returns “yes,”
- otherwise it returns “yes” or “no.” (But we want it to return “no” as much as possible. )

# A candidate for DP

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Suppose we have a complete decision procedure DP' for satisfiability of a formula in two-way CTL.

For predicate  $Q = \bigwedge_i \{\exists \varphi_i\} \wedge \bigwedge_j \{\forall \psi_j\}$ ,

- For  $\varphi_i$ ; if its form is  $\varphi_i = x_i \wedge \varphi'_i$  ( $x_i \in \text{Var}$ ), let  $\varphi'_i := \mathbf{A}_A \mathbf{G} (x_i \rightarrow \varphi'_i)$ , otherwise  $\varphi'_i := \text{true}$ , where  $A$  is the set of all modalities appeared in  $Q$ .

$$\chi_1 := \bigwedge_i \varphi'_i$$

- Let  $\varphi''$  be any of  $\varphi_i$  that is not in the form above. If no such  $\varphi_i$  exists,  $\varphi'' := \text{true}$ .
- $\chi_2 := \bigwedge_j \mathbf{A}_A \mathbf{G} \psi_j$ .

If predicate  $Q$  is satisfiable, then formula  $\chi_1 \wedge \chi_2 \wedge \varphi''$  is satisfiable. (The converse does not hold.)

DP: “Apply DP' to formula  $\chi_1 \wedge \chi_2 \wedge \varphi$ .”

# Problems

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1. If there are lots of  $\exists\varphi$  that cannot be converted to **AG** formulas, precision might be low. (Important information might be lost.)
2. Even if  $Q$  is satisfiable, i.e. it is satisfied by a Kripke structure, it may not be satisfied by a pointer structure.

A Kripke structure  $K(S) = (N, R, \lambda)$  obtained from a pointer structure has properties:

- For each  $x \in \text{Var}$ , there is unique  $n \in N$  such that  $n \models x$ .
- For  $n \in N$  and  $a \in \text{Ptr}$ , there is unique  $m \in N$  such that  $(n, m) \in R(a)$ .

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# Expansion, Closure, ...

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- expansion

- $\text{exp}(\mathbf{E}_A \mathbf{F} \varphi) := \varphi \vee \mathbf{E}_A \mathbf{X} \mathbf{E}_A \mathbf{F} \varphi$
- $\text{exp}(\mathbf{A}_A \mathbf{F} \varphi) := \varphi \vee \mathbf{A}_A \mathbf{X} \mathbf{A}_A \mathbf{F} \varphi$
- $\text{exp}(\mathbf{E}_A \mathbf{G} \varphi) := \varphi \wedge \mathbf{E}_A \mathbf{X} \mathbf{E}_A \mathbf{F} \varphi$
- $\text{exp}(\mathbf{A}_A \mathbf{G} \varphi) := \varphi \wedge \mathbf{A}_A \mathbf{X} \mathbf{A}_A \mathbf{G} \varphi$

- $\varphi \equiv \text{exp}(\varphi)$

- closure

- $\varphi \in \text{cl}(\varphi)$
- $\text{op}(\varphi_1, \dots) \in \text{cl}(\varphi) \implies \varphi_1 \in \text{cl}(\varphi), \dots$   
( $\text{op} = \neg, \wedge, \vee, \mathbf{E}_A \mathbf{X}, \mathbf{A}_A \mathbf{X}, \mathbf{E}_A \mathbf{F}, \mathbf{A}_A \mathbf{F}, \mathbf{E}_A \mathbf{G}, \mathbf{A}_A \mathbf{G}$ )
- $\varphi_1 \in \text{cl}(\varphi), \text{exp}(\varphi_1)$ : defined  $\implies \text{exp}(\varphi_1) \in \text{cl}(\varphi)$

- $\text{Bas}(\varphi) := \{\psi \in \text{cl}(\varphi) \mid \psi \in \text{AP} \text{ or } \psi = \mathbf{E}_A \mathbf{X} \psi' \text{ or } \psi = \mathbf{A}_A \mathbf{X} \psi'\}$

Note: The truth value of  $\psi \in \text{cl}(\varphi)$  can be decided from those of  $\text{Bas}(\varphi)$ .

E.g.  $p \wedge \mathbf{E}_A \mathbf{F} q \equiv p \wedge (q \vee \mathbf{E}_A \mathbf{X} \mathbf{E}_A \mathbf{F} q)$



# $\varphi_0$ -type

---

$\varphi_0$ : the formula the satisfiability of which we will judge

**Type** :=  $\mathcal{P}(\text{Bas}(\varphi_0))$ : the set of  $\varphi_0$ -type.

For  $t \in \text{Type}$  and  $\varphi \in \text{cl}(\varphi_0)$ , we define  $t \Vdash \varphi$ :

- $t \Vdash \varphi \iff \varphi \in t$  (for  $\varphi \in \text{Bas}$ )
- $t \Vdash \neg p \iff p \notin t$  (for  $p \in \text{AP}$ )
- $t \Vdash \varphi \vee \psi \iff t \Vdash \varphi$  or  $t \Vdash \psi$
- $t \Vdash \varphi \wedge \psi \iff t \Vdash \varphi$  and  $t \Vdash \psi$
- $t \Vdash \varphi \iff t \Vdash \text{exp}(\varphi)$  (if  $\text{exp}(\varphi)$  is defined.)

For  $a \in \text{Mod}$ , we define  $R(a) \subseteq \text{Type} \times \text{Type}$ :

$(t, t') \in R(a) \iff$

- $\forall \mathbf{A}_A \mathbf{X} \varphi \in \text{cl}(\varphi_0). (t \Vdash \mathbf{A}_A \mathbf{X} \varphi, a \in A) \implies t' \Vdash \varphi$
- $\forall \mathbf{A}_A \mathbf{X} \varphi \in \text{cl}(\varphi_0). (t' \Vdash \mathbf{A}_A \mathbf{X} \varphi, \bar{a} \in A) \implies t \Vdash \varphi$
- $\forall \mathbf{A}_A \mathbf{F} \varphi \in \text{cl}(\varphi_0). (a \in A, \bar{a} \in A, t \Vdash \mathbf{A}_A \mathbf{F} \varphi, t' \Vdash \mathbf{A}_A \mathbf{F} \varphi) \implies t \Vdash \varphi$  or  $t' \Vdash \varphi$

# Algorithm

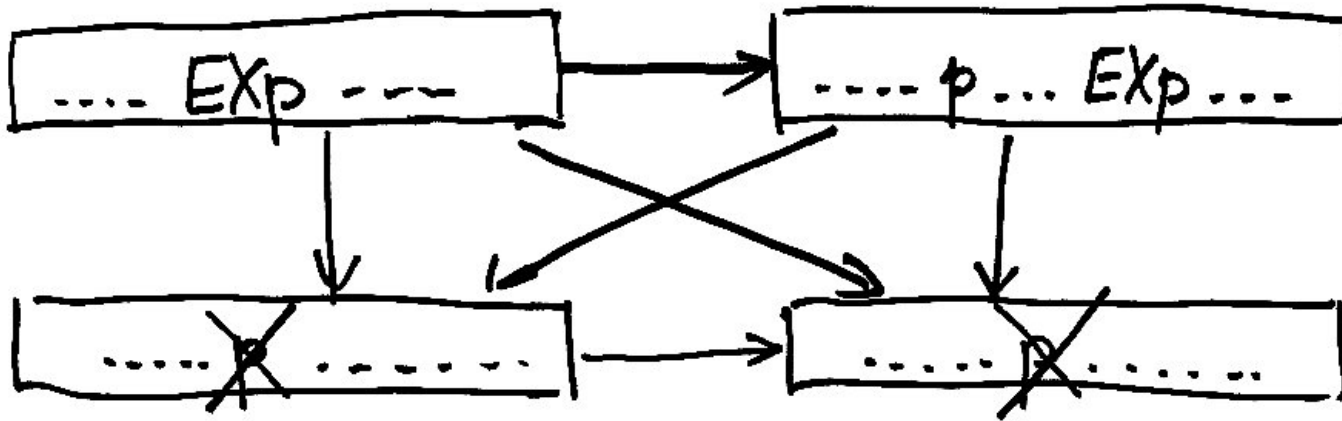
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1. Start with  $T := \text{Type}$ .
2. Remove  $t \in T$  from  $T$  if  $t$  is either EX-inconsistent, EF-inconsistent, or AF-inconsistent in  $T$ .
3. Repeat above as much as possible.
4. If there exists  $t \in T$  such that  $t \Vdash \varphi_0$ , then  $\varphi_0$  is satisfiable, otherwise it is not satisfiable.

# EX-inconsistency

$t \in T$  is EX-inconsistent in  $T$

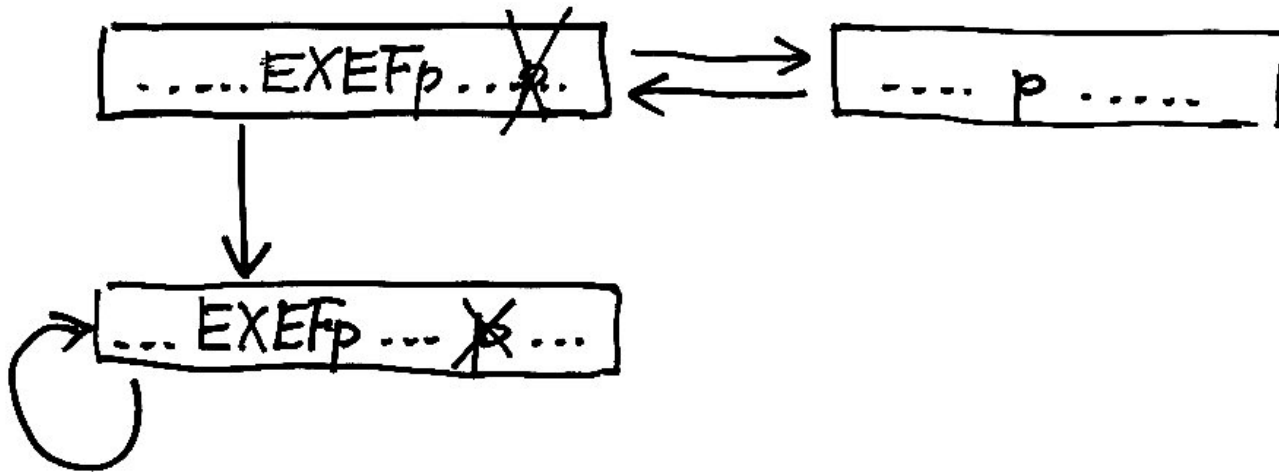
$$\stackrel{\text{def}}{\iff} \exists \mathbf{E}_A \mathbf{X} \varphi \in t. \forall t' \in T. \forall a \in A. (t, t') \in R(a) \implies t' \not\models \varphi.$$



# EF-inconsistency

$t \in T$  is EF-inconsistent in  $T$

$\stackrel{\text{def}}{\iff} \exists \mathbf{E}_A \mathbf{F} \varphi. t \Vdash \mathbf{E}_A \mathbf{F} \varphi, \forall \pi: A\text{-path in } T. \pi_0 = t, \forall i. \pi_i \not\Vdash \varphi.$



# Implementation using BDD

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- Prepare enough BDD variables.
- Foreach BDD variable  $x$ , prepare a BDD variable  $x'$ .
- Foreach  $\varphi \in \text{Bas}(\varphi_0)$ , allocate a BDD variable  $e_F(\varphi)$ .
- For  $t \in \text{Type}$ ,  $e(t) := \bigwedge \{ e_F(\varphi) \mid \varphi \in t \}$   
 $\quad \quad \quad \bigwedge \bigwedge \{ \neg e_F(\varphi) \mid \varphi \in \text{Bas}(\varphi_0) \setminus t \}$
- For  $T \subseteq \text{Type}$ ,  $e(T) := \bigvee \{ e(t) \mid t \in T \}$ .
- For  $R \subseteq T \times T$   
$$e(R) := \bigvee \{ e(t_1) \wedge (e(t_2))' \mid (t_1, t_2) \in R \}$$
  
 $d'$ :  $d$  with all BDD variables replaced with primed one.

# Pseudo Code

```
BDD type, trans[], tabl[], u[];
Boolean satisfiable( $\varphi$ ) {
  type :=  $\bigwedge\{ e_F(\varphi) \rightarrow \bigvee\{ e_M(\varphi, m) \mid m \in \varphi_M \} \mid \varphi \in Cl(\varphi_0)_{EX} \}$ 
  foreach ( $m \in M_0$ ) {
    trans[m] :=  $\bigwedge\{ e_F(\varphi) \rightarrow e'_F(\varphi_X) \mid \varphi \in Cl(\varphi_0)_{AX}, m \in \varphi_M \}$ 
       $\wedge \bigwedge\{ e'_F(\varphi) \rightarrow e_F(\varphi_X) \mid \varphi \in Cl(\varphi_0)_{AX}, \bar{m} \in \varphi_M \}$ 
       $\wedge \neg \bigvee\{ e_F(\varphi) \wedge \neg e_F(\varphi_R) \wedge e'_F(\varphi) \wedge \neg e'_F(\varphi_R) \mid$ 
         $\varphi \in Cl(\varphi_0)_{AU}, m \in \varphi_M, \bar{m} \in \varphi_M \}$ 
    }
  }

  int k := 0;
  tabl[0] := type;
  repeat {
    tabl[k+1] := tabl[k]  $\wedge$  consis(k);
    k := k+1 ;
  } until (tabl[k] == tabl[k-1])
  if (  $e_F(\varphi_0) \wedge$  tabl[k]  $\neq 0$  ) { return true;} else { return false;}
}

BDD consis(k) { return consisEX(k)  $\wedge$  consisEU(k)  $\wedge$  consisAU(k); }

BDD consisEX(k) {
  return  $\bigwedge\{ e_F(\varphi) \wedge e_M(\varphi, m) \rightarrow \exists \vec{x}' ( \text{tabl}[k]' \wedge e'_F(\varphi_X) \wedge \text{trans}[m] ) \mid$ 
     $\varphi \in Cl(\varphi_0)_{EX}, m \in \varphi_M \}$ ;
}
}
```

```

BDD consisEU(k) {
    return  $\bigwedge \{ e_F(\varphi) \rightarrow \text{fulfill}(k, \varphi, \text{stepEU}) \mid \varphi \in \text{Cl}(\varphi_0)_{\text{EU}} \}$ ;
}

BDD consisAU(k) {
    return  $\bigwedge \{ e_F(\varphi) \rightarrow \text{fulfill}(k, \varphi, \text{stepAU}) \mid \varphi \in \text{Cl}(\varphi_0)_{\text{AU}} \}$ ;
}

BDD fulfill(k,  $\varphi$ , step) {
    int j := 0;
    u[0] := tabl[k]  $\wedge e_F(\varphi_R)$ ;
    repeat {
        f[j+1] := f[j]  $\vee ( \text{tabl}[k] \wedge e_F(\varphi) \wedge \neg e_F(\varphi_R) \wedge \text{step}(k, \varphi, j) )$ ;
        j := j+1;
    } until (u[j] == u[j-1])
}

BDD stepEU(k,  $\varphi$ , j) { return  $\exists \vec{x}' (u[j] \wedge \bigvee \{ \text{trans}[m] \wedge e_M(\mathbf{E}_{\varphi_M} \mathbf{X} \varphi, m) \mid m \in \varphi_M \})$  };

BDD stepAU(k,  $\varphi$ , j) {
    return
 $\bigwedge \{ e_F(\psi) \wedge e_M(\psi, m) \rightarrow \exists \vec{x}' (u[j] \wedge e'_F(\psi_X) \wedge \text{trans}[m]) \mid \psi \in \text{Cl}(\varphi_0)_{\text{EX}}, m \in \varphi_M \cap \psi_M \}$ ;
}

```

# Advantages of BDD

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An extreme example:  $\varphi_0 := \bigwedge \{ \mathbf{E}_a \mathbf{F} p_i \mid i = 1, \dots, N \}$

- The size of Type:  $2^N$
- A naive implementation:
  - Creates all  $\varphi_0$ -types and the transition relation at the very first stage.
  - For each  $i$ , all nodes are visited to check EF-inconsistency.
  - $O(N \cdot 2^N)$  time.
- An implementation with BDD:
  - No need to create  $\varphi_0$  types. The number of BDD variables is linear to  $N$ .
  - (For this particular  $\varphi_0$ ,) the size of BDD is also linear to  $N$  during computation.  $O(N)$  time.



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# Hybrid Temporal Logic

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- Formula: two kinds of atomic formulas:
  - **Propositional Variables** (the elements of AP)
  - **Nominals** (the elements of Nom)
- Kripke structure  $K = (S, R, L)$ :
  - $L : AP \cup \text{Nom} \rightarrow \mathcal{P}(S)$
  - For  $n \in \text{Nom}$ ,  $L(n)$  is a singleton.

# Satisfiability

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## Theorem (Sattler, Vardi)

The satisfiability problem of hybrid two-way modal  $\mu$ -calculus is decidable. Its complexity is EXPTIME-complete.

U. Sattler and M. Y. Vardi. The Hybrid  $\mu$ -Calculus. In *Proceedings of the International Joint Conference on Automated Reasoning*, LNAI Vol. 2083, pp. 76–91, 2001.

# Simpler Algorithm for One-way CTL

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$N :=$  the set of nominals appeared in  $\varphi_0$ .

Apply the satisfiability checking regarding nominals as ordinary propositional variables to get  $T = T_0$ .

$\text{EvForm} := \{\varphi \in \text{cl}(\varphi_0) \mid \varphi = \mathbf{E}_A \mathbf{F} \psi \text{ or } \varphi = \mathbf{A}_A \mathbf{F} \psi\}$

Guess two functions  $f$  and  $g$ :

- $f : N \rightarrow T$
- For  $n \in N$ ,  $n \in f(n)$ .
- $g : N \times N \rightarrow \mathcal{P}(\text{EvForm})$
- For  $n_1, n_2 \in N$ ,  $g(n_1, n_2) \subseteq \text{Sat}(f(n_1))$
- $k \geq 2$ ,  $n_1, \dots, n_k \in N$ ,  $n_1 = n_k$   
 $\implies \bigcap \{g(n_j, n_{j+1}) \mid j = 1, \dots, k - 1\} = \emptyset$

Intension:  $f(n)$  is the unique  $\varphi_0$ -type such that  $f(n) \Vdash n$ .  
 $\varphi \in g(n_1, n_2)$  means  $f(n_1)$  can safely rely on  $f(n_2)$  for the eventuality of  $\varphi$ .

Wlog,  $\exists n \in N. f(n) \Vdash \varphi_0$ .

$T' := \{t \in T_0 \mid n \in N, n \in t \implies t = f(n)\}$

For each  $n \in N$ , construct  $T_n$ .

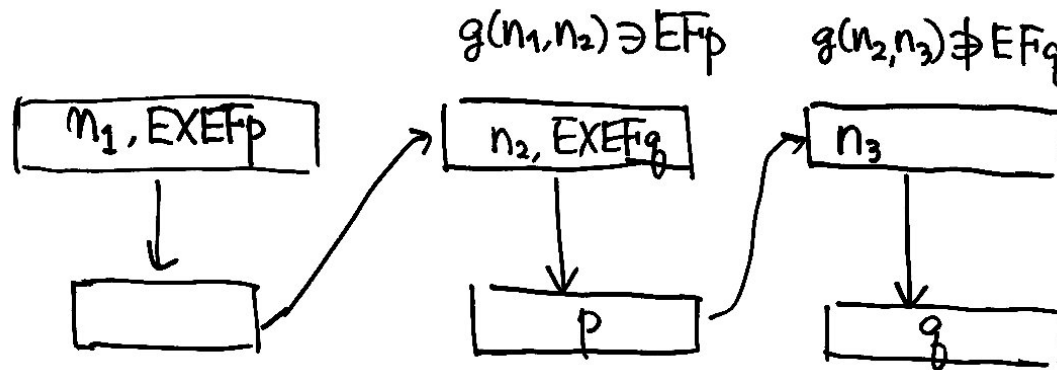
1. Start with  $T_n := T'$ .
2. Remove  $t \in T_n$  from  $T_n$  if  $t$  is either EX-inconsistent, EF-inconsistent, or AF-inconsistent in  $T_n$  with respect to  $n$ .
3. Repeat above as much as possible.
4. If  $f(n)$  remains in  $T_n$ , then the process is all right for  $n$ .

If the process is all right for all  $n \in N$ ,  $\varphi_0$  is satisfiable, otherwise the guess  $(f, g)$  is not correct.

# Consistency with Respect to $n$

Differences are:

- If  $f(n) \neq f(n')$ ,  
 $f(n')$  is always \*-consistent with respect to  $n$ .
- When checking  $\{EF, AF\}$ -consistency with respect to  $n$ ,
  - If  $\varphi \in g(n, n')$ , assume that  $f(n') \Vdash \psi$  holds.
  - If  $\varphi \notin g(n, n')$ , assume that  $f(n') \Vdash \psi$  does not hold.
 where  $\varphi = \mathbf{E}_A \mathbf{F} \psi$  or  $\mathbf{A}_A \mathbf{F} \psi$



# Summary

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- A verification method for software model checking on pointer systems based on predicate abstraction
- Temporal logics ( $+ \alpha$ ) as predicates
- Two key issues:
  - Calculation of preconditions
  - Satisfiability checking
- Implementation with BDD for satisfiability of temporal logic formula
- Hybrid temporal logic